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# A Novel 2-Tuple based Methodology for Deal Recommendation in an E-Business Environment

**Prashant K Gupta**

**Department of Computer Science, South Asian University, New Delhi, India**
***guptaprashant1986@gmail.com***

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**Abstract**

Today, the competition in businesses is increasing rapidly. Thus, more and more businesses are going online, which has given rise to a number of online shopping websites. These websites act as intermediary between the E-Business owners and the consumers. As the customer buys a product online, he/ she is presented with number of deals and the customer selects the best deal among them. The E-Business owners consider numerous criteria while providing a deal to the consumer. However, these criteria are uncertain and their values can be specified linguistically. Elicitation of linguistic information inevitably calls for the use of computing with words (CWW) methodology, to process it. A novel CWW methodology is the 2-tuple fuzzy linguistic approach. It has been applied to a number of application areas, however, to the best of our knowledge, no one has used it for deal recommendation in an E-Business environment. Therefore, in this paper we have proposed a novel 2-tuple based methodology for recommending a deal to a buyer of the product online, by processing the various criteria of the product seller/ E-Business owner. Using the CWW methodology, we generate a numeric score for an e-business owner, which is an indication of the deal quality offered by the owner. We also provide a linguistic recommendation corresponding to the score generated, which may be shown to the buyer. Linguistic recommendation is useful as human beings naturally understand and express themselves using 'words'.

***Keywords:*** *Computing with words; E-Business; Extension principle; symbolic method; Type 1 fuzzy sets; 2-tuple linguistic representation model.*

## I. Introduction

Human beings understand and express themselves naturally using linguistic [24] information. This is true for almost all the real life scenarios. Linguistic information contains 'words' and sentences drawn from natural language [1]. For example, consider a person, who goes to a supermarket to buy a dress. He/ she considers various parameters like color, fit, cost, etc. before making a decision to buy or not any dress. The values of these parameters depend on the person's perception about the dress. Using these perceived values, the person overall evaluates the dress as 'good', 'very good', 'average', etc. Thus, human perception plays an important role in the decision-making, which in this example is to buy the dress or not.

Linguistic information has a remarkable characteristic of being imprecise and vague [1]. Unlike numeric information, which is crisp or precise, linguistic information depends on the subjectivity of user's perceptions. This can be summed up by the adage 'words mean different things to different people' [2]. It is mentioned here that the words such as 'good', 'very good', 'average', etc. stated above, are also called the linguistic terms and collection of such linguistic terms is called a linguistic term set, corresponding to a decision making problem or situation.

To capture and model the uncertainty of 'words' in best possible way, Prof. Zadeh proposed that the concept of fuzzy [25] sets in 1965 and said that the semantics of 'words' be modeled using them [1], [3]. Fuzzy sets are an extension of ordinary or 'crisp' sets. Crisp sets place a degree of affirmation on the belongingness of the objects to the set. Consider an example from [3]. Dogs, cats, etc. will belong to a crisp set defined as collection of animals, and plants, rocks, etc. will not belong to the set. This degree of belongingness of the object is called its membership function (MF). In case of crisp sets, the MF of an object can be either '1' or '0'. No intermediate values are allowed between these two extremes.

However, consider the above example again for set of animals. As stated in [3], the entities like starfish have an ambiguous status with respect to the belongingness to the crisp set of animals. One cannot say with complete affirmation that starfish is an animal. In such scenarios, the collection of objects is represented as fuzzy sets. Fuzzy sets allow varying degree of membership between '0' and '1' viz., the membership function of an object can take any value from the closed interval $[0, 1]$.

The fuzzy sets proposed by Prof. Zadeh in [3], are called the type-1 fuzzy sets (T1 FSs). These sets represent the MF of set objects as crisp or precise values. Later it was realized that the membership value of a fuzzy set could not be precise. This violated the basic principle of uncertainty representation for linguistic information viz., the representation of an uncertain quantity is a precise value. So, Prof. Zadeh proposed the idea of higher-order fuzzy sets of which the most commonly used is the type-2 FSs [1]. Type-2 FSs extend the concept of T1 FSs by providing additional information about each set element, the uncertainty about the membership value of the set element. Some literature on type-2 FSs may also be found in [6], [8]. Prof. Zadeh [1] also conceptualized a special type-2 FS, in which this uncertainty about the membership value are assumed as 1 for all set elements and called such FSs as interval type 2 fuzzy sets (IT2 FSs). Karnik, Mendel and Liang proposed an IT2 fuzzy logic system using the concept of IT2 FSs in 1999 [4]. IT2 FSs have an enhanced capability to capture and model the 'word' uncertainty in a better manner than the T1 FSs. Mendel and Wu also stated this shortcoming of T1 FSs [2].

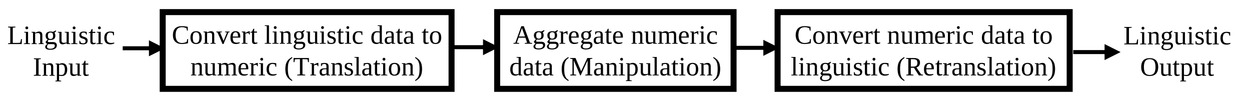


**Figure 1: Computing with words (CWW) methodology [7], [9]**

As stated earlier, human beings understand and express themselves naturally in terms of 'words'. Therefore, to design a computing system that works in a manner similar to the human beings, it is desired that the computing system must process the linguistic information like human beings[1]. For this purpose, Prof. Zadeh proposed a new paradigm and coined the term computing with words (CWW) in 1996 [5]. CWW can facilitate machines to receive human perceptions as input (which are generally expressed as words), manipulate them and generate recommendations. Any system based on CWW performs three basic steps shown in Fig. 1 and is quite useful whenever the linguistic information needs to be processed using a computer. The data input and output to the CWW system are both in linguistic form. Fig. 1 is Yager's representation of CWW [7]. It can be seen that in Yager's CWW methodology, there are three building blocks: translation, manipulation and retranslation.

CWW is a methodology in which words are the object of computation and propositions drawn from natural language, for example 'small', 'large', 'far', 'heavy', 'not very likely', 'extremely', etc. These words activate the CWW system, which converts these words into mathematical representation using the FSs in the first step of translation. This is required because a computer doesn't understand the linguistic information and therefore, it needs to be converted into numerical form. The input information to a CWW system may be from different sources or a single source may provide multiple pieces of information. So, it needs to be aggregated and used for generation of recommendations. This is done in the next step manipulation. The final step is to convert the aggregated numeric information back to the linguistic form because humans do not understand numeric data. Retranslation step does this.

There are different types of CWW methodologies based on this Yager's model. The differentiating parameter for all the methodologies is the way in which the semantics of linguistic information are represented. A popular CWW methodology, that represents the semantics of linguistic information using T-1 FSs, is called extension principle based CWW methodology. Another famous CWW methodology is the symbolic method. It operates on the indices of linguistic terms contained in the linguistic term set. Both the extension principle and symbolic method fail to give unique recommendations in certain situations, which was illustrated in [10]. So a novel CWW methodology called the 2-tuple fuzzy linguistic approach was proposed in [10]. It combines the advantages of both the extension principle and symbolic method. It operates on the indices of the linguistic terms and represents the semantics of linguistic terms using T-1 FS MFs of triangular shape. Since the introduction of the 2-tuple approach, there have been its various developments and applications, a detailed survey about which may be found in [11].

This paper investigates a novel application of 2-tuple based CWW in the deal recommendation in an E-Business environment. Today competition in businesses is increasing rapidly, which can be attributed to numerous factors like globalization, increase in entrepreneurs, etc. As more and more people are entering the business environment, it

[1] Human beings process linguistic information seamlessly due to the capability of human cognitive process.

becomes necessary to stay competitive and make profits. Business processes involves lot of expenditures. Thus, to make the profits business owners try to minimize the expenditures. Also, the new people starting a business may not have large amount of capital to spend on marketing. Fortunately, a number of online websites or platforms such as Amazon, have come in handy for the business owners. These online websites have a number of advantages. These websites provide the business owners an opportunity to market their product [26] to a large number of buyers. The buyer may have the advantage of making a purchase while sitting in the comfort of his/ her office or home, thereby saving the time and inconvenience to going to the market. Another advantage of the website is that the product may be sold to an unknown customer. With all these advantages and many more, not only new or small business owners, but a number of large business owners are also restructuring themselves for e-businesses. In an e-business environment, whenever a buyer purchases a product through an online website, he/ she is presented with a number of offers or deals. Each of these offers/ deals is provided by an e-business owner or a product seller, based on the assessment of various criteria. However, these criteria are uncertain and their values can be expressed in linguistic or in terms of "words". Once the values of these criteria have been expressed in terms of words, they can be processed to recommend a buyer about the best offer, amongst the available ones. Use of linguistic information calls for its processing using the CWW methodology. Thus, in this paper we propose a novel methodology for deal recommendation in an e-business environment. To the best of our knowledge no such application of CWW has been proposed before. Also, the greatest advantage of our proposed approach is that the recommendations about the highest rated deal for the buyer, are expressed in linguistic terms viz., a linguistic term is used to describe the deal. This is because human beings naturally understand and express themselves linguistically2. The interpretability of numeric value is difficult and unnatural to human nature.

In this work, we have also presented a comprehensive and detailed survey of the above stated different CWW approaches viz., extension principle, symbolic method, 2-tuple approach.

Rest of the paper is organized as follows: Section II discusses some literary works that form the base of present work, Section III provides an introduction of CWW and FSs, Section IV provides a description of the extension principle, symbolic method, and the 2-tuple based CWW approach, Section V describes the mathematical details of perceptual computing, Section VI describes the application of various CWW approaches for deal recommendation in an e-business environment and compares the results obtained in each case, and finally Section VII concludes the paper and highlights its future work.

## II. Literature Review

In [20], the authors provided analysis of how fuzzy logic can be applied for the Business Analysis. In [21], the authors presented an outstanding model to classify the web users for the purpose of personalized web search. The model uses the technique of Fuzzy Logic. The member function captured the uncertainty in the user behavior based on factors that affected the user's interest. In the present work, the processing has been done on "words" determining the profitability of the deal for the targeted buyer as well as the dealer in the retail environment. In [22], a model for achieving the personalization of the web search for the user was presented by processing the user's web navigation patterns. The processing of the humungous information retrieved was done based on the fuzzy membership functions based on the user psychology. The psychology of the user was made the basis of personalizing the content on the web, which is a unique way of classification of the users. The same approach has been used in our paper, but the parameters that govern the classification are the parameters that determine the profitability of the business deals in the e-commerce environment. In [23], the authors have reported the issues with the recommendation systems used today in e-service environment. In the paper, they presented a technique which dealt with the tree structure of the user profile using fuzzy preferences for the generation of recommendation. The approach had an edge over the recommendation systems that made use of crisp data for determining the user preferences because user preferences are generally vague. But, the users express their preferences in the form of "words", therefore in our system we generate personalized recommendation for the business dealer by processing these "words" such that the deal ensuring the maximum profit gets the highest score.

## III. Fuzzy Sets and CWW

Prof. Zadeh proposed the idea of CWW in [5]. It provides a mapping between linguistic and numeric information. Real life systems involve linguistic information and opposed to

[2] A shorter version of the present work has been presented at CIMSIM 2015 [16].

it, computers understand numbers. So the use of CWW becomes inadvertently important.

Linguistic data is however uncertain because 'words mean different things to different people' [42]. So to model the linguistic data, Prof. Zadeh proposed the concept of fuzzy sets in 1965 [43]. They are an extension of crisp sets and they are more suited to model the data in real life applications. For example, we conducted a survey based on the data taken from a website that depicted the world university rankings, in a country. To the best of our knowledge, the performance of the university was given in 5 star ratings and the highest rating achieved by a university was 4.5/5. When we asked the students whether they considered 4.5 to be a good or bad rating, we found that students gave varying feedback between these two extremes viz. students considered rating to be very good, moderately good, fair, etc. Though the overall rating is precise numeric value of 4.5, every user has a different interpretation for the same. This is an example of uncertainty inherent in the linguistic data and is best expressed by fuzzy sets.

Every element belonging to the fuzzy sets given by Zadeh in [1] is expressed mathematically as a combination of two terms: $(x, \mu_x)$ where x is the element belonging to the set and $\mu_x \in [0, 1]$ is its membership value or MF. MF gives the idea of degree of belongingness x to the fuzzy set. Commonly chosen scale is $0$ to $1$ but any scale can be chosen. This is opposed to crisp sets where $\mu_x \in \{0, 1\}$, 0 being exclusion from the set and 1 being the inclusion. These were later called T1 FSs and their MFs were shown in triangular form.

The membership function of a T1 FS is crisp which is contrary to the very definition of fuzzy sets where one cannot be sure of something which is fuzzy. Therefore, to account for the uncertainty about the MF, concept of type 2 FSs was proposed later. They are denoted as $(x, \mu_x, J_x)$, where the third variable $J_x$ is added to the T1 FS representation and denotes the uncertainty that one has about MF $\mu_x$. In such sets, $\mu_x$ is called the primary MF and the $J_x$ is called secondary MF. Mendel et. al. considered the value of $J_x = 1$ everywhere and such FSs are called IT2 FSs. They are shown graphically in Fig. 2.

In Fig. 2, the quantitity on the y-axis is the membership value (u) and on the x-axis is the variable. The UMF stands for Upper membership function and LMF stands for lower membership function. Both the UMF and LMF are T1 FSs.

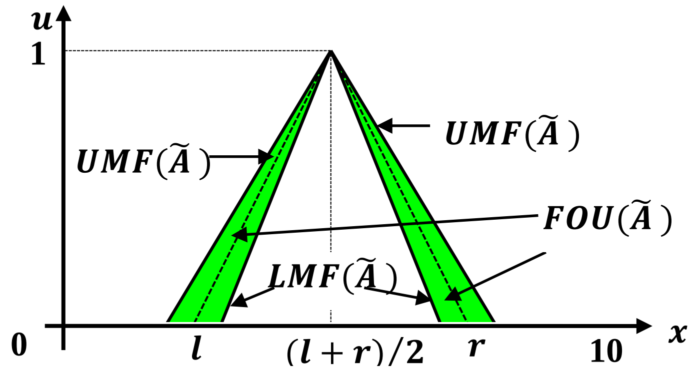


**Figure 2: Graphical representation of IT2 FS**

The shaded region in the Fig. is called the Footprint of Uncertainty (FOU). The FOU is bound from above by the UMF and below by the LMF.

In the IT2 FSs, since the secondary membership value is 1 everywhere, therefore the spread of FOU gives an idea of the uncertainty captured by the IT2 FS model. The FOU can be visualized to be sitting atop the primary MF of the word that the IT2 FS models. There is a dashed line or T1 FS depicted in Fig. 2, whose left and right ends rest on the x-axis at l and r, respectivley. It's centre at ${(l + r)}/{2}$ has a membership value of 1. This T1 FS is called an embedded T1 FS. The FOU can be assumed to be a union of continuum of such embedded T1 FSs.

Fuzzy sets capture and model the word uncertainties. This is quite close to human cognitive process. Therefore, to design any computing system that can function similar to human beings, it should have the capability to process 'words'. Prof. Zadeh gave the concept of CWW methodology to design such systems [5]. The basic principle underlying a CWW methodology is the one to one mapping between the linguistic and the numeric information. Numerous ways have been proposed for CWW based on the linguistic information representation viz. based on T1 FSs, symbolic method or IT2 FSs. One famous T1 FS based representation is the extension principle. Symbolic method operates on the indices of the linguistic terms in the term set. 2-tuple fuzzy linguistic representation model combines aspects of both extension principle and symbolic method. Mendel et al. proposed IT2 FS based CWW and has been given the special name of perceptual computing. The details of all these CWW techniques are discussed in subsequent sections.

## IV. Extension Principle, Symbolic Method And 2-Tuple Based CWW Methodologies

In this section we will discuss the details of extension principle, symbolic method and 2-tuple based CWW methodologies. For details, see [10].

### A. Extension principle

The extension principle represents the linguistic terms of the term set in the form of triangular T1 MF and each term as a collection of three values: $(l, c, r)$, l being the left end of the traingle, c it's center and r being the right end. Computations are performed on each value and recommendations are generated. Consider a set of linguistic terms $S = \{s_0, s_1, \dots \dots .., s_g\}$. The cardinality of set is $g + 1$. Each term is represented by a triangular membership function centered at ${}^{i}/_{g}$, $i = 0$ to g. For example, in a real life problem we are representing the linguistic variable 'temperature' that can take linguistic values 'very cold (vc)', 'cold (c)', 'moderate (m)', 'hot (h)' and 'very hot (vh)'. Other linguistic values are also possible but we have chosen only these five for simplicity of discussion. They are represented by triangular MFs and are shown in Fig. 3, with each triangular T1 MFs centered at ${}^{i}/_{4}$, $i = 0$ to 4. Let the feedback provided by three people for the day's temperature be 'very cold', 'cold' and 'moderate', respectively. To aggregate these feedback values, each feedback linguistic variable is initially reporesented as collection of three values as: $vc = \{0, 0, 0.25\}, c = \{0, 0.25, 0.5\}$ and $m = \{0.25, 0.5, 0.75\}$. These collective performance vector is given as $C = \{0.08, 0.25, 0.5\}$.

For recommending a linguistic recommendation, the distance of the collective performance vector from each of the linguistic terms in the term set is found using the values for parameters $P_1, P_2$ and $P_3$ to be 0.2, 0.6 and 0.2 respectively. Computing the distances, we get the minimum distance occurs from the linguistic term cold. Thus, the temperature of the day is 'cold'.

### B. Symbolic method

Symbolic method operates on the indices of linguistic terms in the term set. Let again, the linguistic terms of a term set are represented as: $S = \{s_i | i = 0 \text{ to } g\}, g + 1$ being the cardinality of the set. The method assigns a weight from the weight vector $W = [w_1, \dots \dots \dots \dots w_i]$; corresponding to each of the ith information term; such that each of $w_p \in [0,1]; p = 1$ to i and $\sum_{p=1}^{i} w_p = 1$. The technique first orders the linguistic term set containing the linguistic feedback and then aggregates the data values according to the function $(SM^j)$ given in Eqs. (1)-(2):

For j>2,

$$SM^j\{w_k, s_k, k = 1, .. j\} = (w_1 \odot s_1) \oplus \left((1 - w_1) \odot SM^{j-1}\{\sqcap_h, s_h, h = 2, .. j\}\right) \quad (1)$$

where $\sqcap_h = {}^{w_h}/_{\sum_{l=2}^{j} w_l}$ ; $h = 2, 3, \dots \dots \dots \dots j$

For j=2,

$$SM^2\left\{\{w_1, 1 - w_1\}, \{s_l, s_q\}\right\} = (w_1 \odot s_l) \oplus \left(1 - w_1 \odot s_q\right) = s_r \quad (2)$$

such that $r = \min\{g, q + \text{round}(w_1.(q - l))\}$; $g + 1$ being the cardinality of the set to be aggregated and round() is the usual round function. It can be seen that aggregation function performs convex combination of information at each step proceeding in top-down manner. At each step of the aggregation, computations are performed on the numeric indexes of the term set to give an aggregated value as $S^{SM}$. Finally, the recommended value is a unique index of the term belonging to the term set S.

Consider the problem of previous subsection again. The linguistic term set for variable 'temperature', is given as:

$$S_{Temperature} = \{s_0: \text{very cold}, s_1: \text{cold}, s_2: \text{moderate}, s_3: \text{hot}, s_4: \text{very hot}\} \quad (3)$$

Here each linguitic term, $s_i$, occurs at index $i, i = 0, \dots 4$. Let the feedback provided by three people for the day's temperature be '$s_0$: very cold', '$s_1$: cold' and '$s_2$: moderate', respectively. For the purpose of illustration, we consider equal weights for all the terms. thus, weight matrix is given as:

$$W = \left[w_1 = \frac{1}{3}, w_2 = \frac{1}{3}, w_3 = \frac{1}{3}\right] \quad (4)$$

In the first step, the terms are ordered as:

$$S = \{s_2, s_1, s_0\} \quad (5)$$

Using Eqs. (1)-(2), we get:

$$SM^3\{w_k, s_k, k = 1 \dots 3\} = \left(\frac{1}{3} \odot s_2\right) \oplus \left(\frac{2}{3} \odot SM^2\{\sqcap_h, s_h, h = 2, 3\}\right) \quad (6)$$

Where $\sqcap_h = \frac{1}{2}$, each $w_k, k = 1 \dots 3$, is a term drawn from W, shown in Eq. (4) and each $s_k, k = 1 \dots 3$, is a term drawn from S, shown in Eq. (5).

For j=2,

$$SM^2\left\{\left\{\frac{1}{2},\frac{1}{2}\right\},\{s_1,s_0\}\right\} = \left(\frac{1}{2} \odot s_1\right) \oplus \left(\frac{1}{2} \odot s_0\right) = s_r \quad (7)$$

Here $r = \min(3,1) = 1$. Therefore value from Eq. (7) is $s_1$. Putting this in Eq. (6), we get

$$SM^3\{w_k, s_k, k = 1 \dots 3\} = \left(\frac{1}{3} \odot s_2\right) \oplus \left(\frac{2}{3} \odot s_1\right) = s_r \quad (8)$$

Value of r in Eq. (8), is given as: $r = \min(3,1) = 1$. Thus, recommended linguistic term is $s_1 = \text{cold}$. Thus, the temperature of the day is ‘cold’.

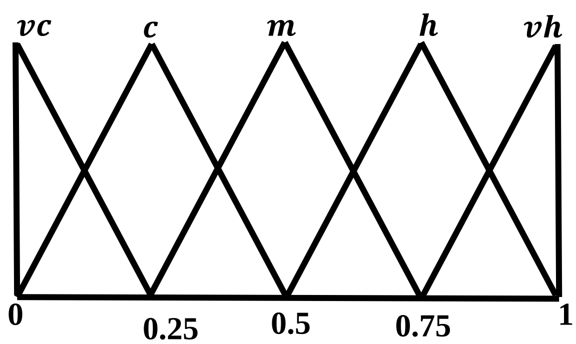


**Figure 3: Triangular membership function representation of ‘temperature’**

## C. 2-Tuple based CWW methodology

2-tuple approach for CWW is inspired from both the extension principle and the symbolic method. Here, the linguistic terms used to represent the linguistic information are distributed uniformly on the scale of 0 to 1. Consider a set of linguistic terms $S = \{s_0, s_1, \dots \dots .., s_g\}$. The cardinality of set is $g + 1$. In any decision making problem, lets say the result of aggregation of preferences obtained from multiple experts is $\beta$. The recommended solution is given by $(s_j, \alpha)$ where $j = \text{round}(\beta), \alpha = \beta - j$. Here round is the usual mathematical rounding operation and $\alpha \in [-0.5, 0.5)$ is called the translation distance. For example, the result of aggregation in a decision problem is 2.3. So, $\beta = 2.3$, $\text{round}(2.3) = 2, \alpha = 2.3 - 2 = 0.3$. The solution is given by $(s_2, 0.3)$. Thus, each value in 2-Tuple representation is twin valued: the linguistic term and the translation distance. Translation distance gives a measure of the difference between the index of actual term $s_j \in S$ and the aggregated value. Therefore by corollary, for $s_j \in S$, 2-Tuple representation becomes $(s_j, 0)$.

Considering again the problem of the feedback provided by three people for the day’s temperature be ‘very cold’, ‘cold’ and ‘moderate’, respectively. To aggregate these feedback values, using 2-tuple approach, each linguistic feedback value is represented by its index as: ‘$s_0$: very cold’, ‘$s_1$: cold’ and ‘$s_2$: moderate’, respectively. These data values are aggregated using the weighted average operator of 2-tuple approach. this aggregation is shown in Eq. (9) as:

$$\beta = \frac{0 + 1 + 2}{3} = 1 \quad (9)$$

$$j = \text{round}\ (\beta) = \text{round}\ (1) = 1 \quad (10)$$

$$\alpha = \beta - \text{round}\ (\beta) = 1 - 1 = 0 \quad (11)$$

Therefore, the recommended linguistic term is given as: $(s_j, \alpha)$ where $j = \text{round}(\beta), \alpha = \beta - j$. This means, $(s_j, \alpha) = (s_1, 0) = \text{cold}$. Thus, the temperature of the day is ‘cold’.

# V. CWW for Deal Recommendation in E-Business Environment

Today competition in businesses is increasing rapidly as more and more people are entering the business environment. It thus becomes necessary to stay competitive and make profits. Increasing provides can also occur by cutting down on the wasteful expenditures. With a number of online websites or platforms such as Amazon, emerging as an interface between the buyer and the business owners, a number of business owners are restructuring themselves for e-businesses. In an e-business environment, whenever a buyer purchases a product through an online website, he/ she is presented with a number of offers or deals. Each of these offers/ deals is provided by an e-business owner or a product seller, based on the assessment of various criteria. However, we identified that there are certain criteria for which the values can be expressed in linguistic form or in terms of “words”. These linguistic values can be processed to recommend the best offer to a buyer, from amongst the available ones.

Use of linguistic information calls for its processing using the CWW methodology and generating recommendations. Use of CWW methodology offers two advantages. Firstly, the methodology generates a numeric score for the deal or offer provided by the respective e-business owner. These scores can be used to rank the deals and the best deal can be recommended to the buyer. Secondly, a linguistic value is generated corresponding to this numeric score. As human beings naturally understand and express themselves using linguistic information, therefore, linguistic recommendations are quite useful. Using the linguistic recommendation, the user can assess the deal qualitatively.

In this work, each of the criteria used for deal recommendation in an e-business environment, act as linguistic variables (or linguistic term sets) and have associated linguistic values. These are listed in Table I. One value is selected from each term set corresponding to each criteria. These values are processed by the technique of CWW to generate recommendations, which are also shown in Table I. We have solved the problem of deal recommendation in an e-business environment using various CWW techniques viz., extension principle, symbolic method, 2-tuple approach and the perceptual computing. We also presented the comparison of results obtained in all three techniques. It is mentioned here that an initial work in the direction was [16]. Inspied from this work, we modified some of the parameters and the linguistic values used for the deal recommendation in e-business environment.

**TABLE I: CRITERIA AND RECOMMENDATIONS ALONG WITH THEIR LINGUISTIC VALUES**

| | Criteria/ | Linguistic values |
|---|---|---|
| Criteria | Product Cost (C) | Very Little (CVL) |
| | | Low (CL) |
| | | Moderate(CM) |
| | | High (CH) |
| | | Very High (CVH) |
| | E-Business Owner's Experience/ Seller Experience (SE) | Novice (SEN) |
| | | Limited Experience (SELE) |
| | | Moderate (SEM) |
| | | Large (SEL) |
| | | Expert (SEE) |
| | Seller's feedback (SF) | Very Poor (SFVP) |
| | | Poor (SFP) |
| | | Average (SFA) |
| | | Good (SFG) |
| | | Very Good (SFVG) |
| | Seller's profit (SP) | Very Less (SPVL) |
| | | Less (SPL) |
| | | Moderate (SPM) |
| | | High (SPH) |
| | | Very High (SPVH) |
| | Inventory level (I) | Negligible (IN) |
| | | Low (IL) |
| | | Adequate (IA) |
| | | High (IH) |
| | | Very High (IVH) |
| | Production Time (PT) | Very Less (PTVL) |
| | | Less (PTLE) |
| | | Adequate (PTA) |
| | | Large (PTL) |
| | | Very Large (PTVL) |
| | Priority of deal with respect to other deals (P) | Very Low (PVL) |
| | | Low (PL) |
| | | Moderate (PM) |
| | | High (PH) |
| | | Very High (PVH) |
| Recommendation | Deal type (D) | Not Good (DNG) |
| | | Below Average (DBA) |
| | | Average (DA) |
| | | Good (DG) |
| | | Very Good (DVG) |

**TABLE II: CRITERIA VALUES OF TWO E-BUSINESS OWNERS**

| Parameter | Criteria values | |
|---|---|---|
| | **Owner 1 (O1)** | **Owner 2 (O2)** |
| Product Cost (C) | Low | Moderate |
| E-Business Owner's Experience/ Seller Experience (SE) | Large | Moderate |
| Seller's feedback (SF) | Average | Good |
| Seller's profit (SP) | Moderate | Less |
| Inventory level (I) | Adequate | Low |
| Production Time (PT) | Adequate | Large |
| Priority of deal with respect to other deals (P) | Moderate | Moderate |

**TABLE III: LINGUISTIC FEEDBACK OF TWO OWNERS IN 2-TUPLE FORM**

| Parameter | Criteria Values | | | |
|---|---|---|---|---|
| | Owner 1 (O1) | | Owner 2 (O2) | |
| | Linguistic feedback | 2-tuple representation | Linguistic feedback | 2-tuple representation |
| Product Cost (C) | Low | $(s_1, 0)$ | Moderate | $(s_2, 0)$ |
| E-Business Owner's Experience/ Seller Experience (SE) | Large | $(s_3, 0)$ | Moderate | $(s_2, 0)$ |
| Seller's feedback (SF) | Average | $(s_2, 0)$ | Good | $(s_3, 0)$ |
| Seller's profit (SP) | Moderate | $(s_2, 0)$ | Less | $(s_1, 0)$ |
| Inventory level (I) | Adequate | $(s_2, 0)$ | Low | $(s_1, 0)$ |
| Production Time (PT) | Adequate | $(s_2, 0)$ | Large | $(s_3, 0)$ |
| Priority of deal with respect to other deals (P) | Moderate | $(s_2, 0)$ | Moderate | $(s_2, 0)$ |

Consider the values for the criteria of two e-business owners, shown in Table II. Next we process the values of these criteria using each of the CWW technique to generate recommendations, about the quality of deal referred to the buyer.

For processing the criteria values of owners using the 2-tuple approach, we consider the feedback of owners shown in Table IV. This is converted to 2-tuple form, by making the translation distance 0, because these terms are directly drawn from the term set. Thus, the criteria values of owners in 2-tuple form is shown in Table V as:

Once the criteria values of owners is been represented in the form shown in Table V, it is aggregated. Consider criteria values of owner 1 (O1). Following the aggregation approach similar to Section IV for 2-tuple approach we get,

$$\beta = \frac{1+3+2+2+2+2+2}{7} = 2 \qquad (24)$$

$$\mathrm{j} = \mathrm{round}\ (\beta) = \mathrm{round}\ (2) = 2 \qquad (25)$$

$$\alpha = \beta - \mathrm{round}\ (\beta) = 2 - 2 = 0 \qquad (26)$$

Therefore, the recommended linguistic term is given as: $(s_j, \alpha)$ where $j = \mathrm{round}(\beta), \alpha = \beta - j$. This means, $(s_j, \alpha) = (s_2, 0) = (da, 0)$ or 'average'. Proceeding similarly for owner 2 (O2), the deal value is also found to be 'average'. Thus, the same recommendation is generated in both cases.

## VI. Conclusion And Future Work

CWW is a novel mathematical technique that propvides a one-to-one mapping between linguistic and numeric information. Linguistic information is generated in human driven systems and has in-built uncertainty. So the use of CWW to handle such information becomes indispensable. Linguistic information is best modeled by fuzzy sets. In this paper, we have presented a comprehensive and detailed survey of CWW approaches based on type-1 FSs, symbolic method, 2-tuple approach and interval type 2 FSs. We have applied these CWW approaches to the problem of deal recommendation in an e-business environement.

We feel that in the current scenario of increasing competition in businesses, a large number of businesses are orienting themselves as e-businesses. Simultaneously a number of online shopping websites are becoming common. These websites act as an interface between the different e-business owneres and the buyers. Whenever a buyer browses an online shopping website to buy a product, he/ she is always presented with a number of deals offered by various e-businesse owners. However, the deal offered by the e-business owner is based on the evaluation of numerous criteria, which are uncertain. We found that these criteria can take linguistic values. Linguistic data calls for the novel technique of CWW for data processing. Therefore, in the present work, we have used the CWW techniques for the problem of selecetion of best deal, out of various ones offered by various e-business owners.

We considered the test case of two e-business owners for comparative analysis of the deals, offered by the two corresponding to the same product. The results obatined with all the CWW techniques are summarized in Table VIII. From the Table it can be seen that IT2 FS based CWW technique viz., perceptual computing gives unique results whereas other techniques fail to do so. Furthermore, the numeric values of the deal scores can be used to recommending the best deal to the targeted buyer. We have also generated linguistic recommendations, which can be useful for the buyers.

## References

[1] L. A. Zadeh. "The concept of a linguistic variable and its application to approximate reasoning", Part I. Information Sciences, vol. 8, no. 3, pp.199–249, 1975.

[2] J. M. Mendel, D.Wu, "Perceptual Computing: Aiding People in Making Subjective Judgments", Wiley, IEEE Press, 2010.

[3] L. A. Zadeh, "Fuzzy sets", Information and control, vol. 8, pp 338-353, 1965

[4] N. N. Karnik, J. M. Mendel and Q. Liang, "Type-2 Fuzzy logic systems", IEEE Transactions on Fuzzy systems, vol. 7, no. 6 , pp. 643-658, 1999

[5] L. A. Zadeh, "fuzzy logic=computing with words", IEEE transactions on fuzzy systems, vol. 4,may1996, pp.103-111

[6] J. Mendel, "Type-2 fuzzy sets: Some questions and answers," IEEE Connections, vol. 1, pp. 10–13, 2003.

[7] R.R. Yager, "Computing with words and information/ intelligent systems 2:applications", Chapter approximate reasoning as a basis for computing with words, physica verlag, 1999, pp 50-77

[8] S. Greenfield, and R. John, "The uncertainty associated with a type-2 fuzzy set," in Views on Fuzzy Sets and Systems from Different Perspectives Philosophy and Logic, Criticisms and Applications, Studies in Fuzziness and Soft Computing, vol. 243. New York, NY, USA: Springer-Verlag, 2009, pp. 471–483.

[9] J.M. Mendel, "Computing with words: Zadeh, Turing, Popper and Occam", IEEE COMPUTATIONAL INTELLIGENCE MAGAZINE, Nov. 2007

[10] F. Herrera, L. Martinez, "A 2-Tuple Fuzzy Linguistic Representation model for computing with words", IEEE Trans. Fuzzy Syst., vol. 8, no. 6, pp. 746–752, 2000

[11] L. Martinez, F. Herrera, "An overview on the 2-tuple linguistic model for computing with words in decision making: extensions, applications and challenges", Information Sciences, vol. 207, pp. 1-18, 2012

[12] J. M. Mendel, D. Wu, "Perceptual reasoning for perceptual computing", IEEE Transactions on Fuzzy Systems, vol. 16, no. 6, pp. 1550-1564, 2008

[13] P. K. Gupta, P. K. Muhuri, "Perceptual Computing Based Performance Control Mechanism for Power Efficiency in Mobile Embedded Systems", (FUZZ-IEEE), 2014.

[14] P. K. Gupta, P. K. Muhuri, "Per-C based green computing model for handheld devices: An application of single person FOU", IEEE International Conference on SMC, 2016

[15] P. K. Muhuri, P. K. Gupta, J. M. Mendel, "User Satisfaction-Aware Power Management in Mobile Devices based on Perceptual Computing", IEEE Transactions on Fuzzy Systems, DOI 10.1109/TFUZZ.2017.2773020

[16] P.K. Gupta, M. Madan, C. Kapoor, "Perceptual Computing Based Profit Generation Advisor for the Business Decision Making in E-Commerce", 2015 7th IEEE CIMSim International Conference on. Computational Intelligence, Modelling and Simulation (CIMSim), 2015, pp 1-6.

[17] F. Liu and J. M. Mendel, "Encoding Words Into Interval Type-2 Fuzzy Sets Using an Interval Approach", IEEE Transactions on Fuzzy Systems, vol 16, no. 6, pp.1503-1521, 2008

[18] D. Wu, J. M. Mendel and S. Coupland. "Enhanced interval approach for encoding words into interval type-2 fuzzy sets and its convergence analysis", IEEE Transactions on Fuzzy Systems, vol. 20, no. 3, pp. 499-513, 2012

[19] M. Hao and J. M. Mendel. "Encoding words into normal interval type-2 fuzzy sets: HM approach", IEEE Transactions on Fuzzy Systems, vol. 24, no. 4, pp. 865-879, 2016.

[20] Nikola Gradojevic,Ramazan Gencay, "Fuzzy Logic, trading uncertainty and technical trading", 2012, Elsevier

[21] S. Sendhilkumar, K. Selvakumar, G. S. Mahalakshmi, "Application of Fuzzy Logic User Classification in Personalized Web Search", International Journal on Cybernetics & Informatics, Vol. 3, No. 3, June 2014

[22] Sachin Agarwal, Pallavi Agarwal, "A Fuzzy Logic Approach to Search Results' Personalization by Tracking User's Web Navigation Pattern and Psychology", 17th IEEE Conference on Tools with Artificial Intelligence, 2005

[23] Dianshuang Wu, Guangquan Zhang, Jie Lu, "A Fuzzy Preference Tree-Based Recommender System for Personalized Business-to-Business E-Services", IEEE Transactions on Fuzzy Systems, 2014

[24] Pinto, A., Oliveira, H. G., Figueira, Á., & Alves, A. O. (2017). Predicting the Relevance of Social Media Posts Based on Linguistic Features and Journalistic Criteria. New Generation Computing, 35(4), 451-472.

[25] Almendros-Jiménez, J. M., Tedesqui, A. L., & Moreno, G. (2015). Fuzzy xpath through fuzzy logic programming. New Generation Computing, 33(2), 173-209.

[26] Baccianella, S., Esuli, A., & Sebastiani, F. (2013). StarTrack: the next generation (of product review management tools). New generation computing, 31(1), 47-70.