A Survey on Direction of Arrival Estimation Techniques

Deepali¹ and Sanjay Tomar²
¹Research Scholar, PDM College of Engineering, Bahadurgarh
²Asst. Professor, PDM College of Engineering, Bahadurgarh
¹deepali.1945@gmail.com

Abstract
Direction of Arrival (DOA) estimation methods are useful in Sonar, Radar and Advanced Satellite and Cellular Communication systems. DOA estimation addresses the problem of locating sources which are radiating energy that is received by an array of sensors with known spatial positions. It was the desire to locate and track enemy aircraft using radar that initiated the concept of sensor array processing as a sub-discipline of electrical engineering in the 1940s. The problem of direction finding or direction-of-arrival (DOA) estimation has become important in many other fields besides radar, for example, acoustic signals received by an array of hydrophones are used in underwater applications to detect and locate submarines and surface vessels. The wide range of applications for source localization has induced a correspondingly large amount of research and refinement of techniques for estimating source locations. Over the years, these array processing techniques have become more essential for battlefield and situational awareness. This paper studies the concept of direction of arrival estimation and the sparse techniques.

Keywords: DOA, W-CMSR, Sparse, Covariance Matrix.

I. Introduction
Direction of arrival estimation is an active research area in radar, sonar navigation, geophysics and acoustic tracking. General DOA estimation addresses the problem of locating sources which are radiating energy that is received by an array of sensors with known spatial positions. Wideband signals are widely used in various radar and sonar systems, and many methods have been proposed to estimate their directions-of-arrival (DOA) [1]. Most of those methods decompose the incident wideband signals into narrowband components, and then realize wideband DOA estimation with incoherent or coherent techniques. However, there are two significant disadvantages within these kind of methods. First, DOA pre-estimates of the incident signals are required for spectral focusing, and the precision of those pre-estimates largely influences the performance of DOA estimation. Second, they need the a priori information of the incident signal number which may not be available, especially in the non-cooperative scenarios [2]. Therefore, substitutive methods are required to explore better solutions for the problem of wideband DOA estimation. Recently there has been a great interest in estimating the DOAs for wideband sources, whose energy is spread over a broad bandwidth. For example, acoustic signals can range from 20 Hz to 20 KHz depending upon the type of source. For estimating the direction of arrival of wideband sources many of the narrowband DOA estimation algorithms can be directly applied. An intuitive way of generalizing a narrowband algorithms to wideband algorithm is to use the discrete fourier transform (DFT) to decompose the signal into narrowband signals of different frequencies and apply narrowband algorithms to each component, and fuse the overall estimation results. But the better estimation accuracy is usually obtained when applying wideband algorithms to wideband sources.

The technique of sparse representation provides a new perspective for DOA estimation. Methods of this category recover the spatial distribution of the incident signals by directly representing the array output on an over-complete dictionary under sparsity constraint, and the prior information of signal number is no longer a necessity. There are many methods that addresses sparsity-based DOA estimation. The first one, as far as we know, is the global matched filter (GMF) method, GMF bases on uniform circular arrays, and exploits the beamformer samples to realize DOA estimation [3]. Then L1-SVD method was proposed, which reduces the dimension of the observations via singular decomposition. L1-norm-based optimization model achieved great success in those two methods. But to be more rigorous, the sparsity constraint on the spatial distribution of the incident signals should be modeled with L0-norm, so that the model parsimony is directly concerned by the
sparsity constraint. However the L0-norm-based optimization problem is not globally convergent, and it is NP-hard to solve it directly, so convex approximation is introduced and L1-norm is used in the above two methods to facilitate the optimization process. More recently joint sparse approximation technique was introduced for DOA estimation, in which, the L0-norm constraint is approached by a family of convex functions, and a method called JLZA-DOA is proposed. Although satisfying performance was reported for JLZA-DOA, global convergence is not guaranteed. L1-SVD and JLZA-DOA have also been extended to wideband signals. However, wideband L1-SVD and JLZA-DOA decompose the array output into narrowband components and obtain spectra in discrete frequency bins, so further efforts are required to derive integrative wideband DOA estimates from them [4].

II. Sparse Representation Techniques

Sparse representations are the representations that account for most or all information of a signal with a linear combination of a small number of elementary signals called atoms [5]. Often, the atoms are chosen from a so called over-complete dictionary. A new direction-of-arrival (DOA) estimation method is proposed based on sparse representation of array covariance vectors in which DOA estimation is achieved by jointly finding the sparsest coefficients of the array covariance vectors in an over complete basis. It means that given a signal, firstly we form the dictionary which contains the atoms that represent the signal than we find the smallest set of atoms from the dictionary to represent the signal. The proposed method not only has high resolution and the capability of estimating coherent signals based on an arbitrary array, but also gives an explicit error-suppression criterion that makes it statistically robust even in low signal-to-noise-ratio (SNR) cases [6, 7].

a. Covariance Matrix

It is a matrix whose elements in \((i, j)\)th position is the covariance between \(i\)th and \(j\)th element of random vector (array output of sensors). Intuitively, the covariance matrix generalizes the notion variance to multiple dimensions [8].

\[
\text{Cov}(X_{i}, Y_{j}) = E[(X_{i} - \mu_{i})(X_{j} - \mu_{j})]
\]

And the covariance matrix can be formed as:

\[
\begin{align*}
\sum = \\
E[(X_{1} - \mu_{1})(X_{1} - \mu_{1})] & \quad E[(X_{1} - \mu_{1})(X_{2} - \mu_{2})] & \ldots & \quad E[(X_{1} - \mu_{1})(X_{n} - \mu_{n})] \\
E[(X_{2} - \mu_{2})(X_{1} - \mu_{1})] & \quad E[(X_{2} - \mu_{2})(X_{2} - \mu_{2})] & \ldots & \quad E[(X_{2} - \mu_{2})(X_{n} - \mu_{n})] \\
& \vdots & \ddots & \vdots \\
E[(X_{n} - \mu_{n})(X_{1} - \mu_{1})] & \quad E[(X_{n} - \mu_{n})(X_{2} - \mu_{2})] & \ldots & \quad E[(X_{n} - \mu_{n})(X_{n} - \mu_{n})]
\end{align*}
\]

b. Several Sparse Representation Techniques

W-CMSR, L1-SVD, JLZA-DOA are some of the well known sparse representation techniques used for the direction of arrival estimation. These methods are the successful attempt to estimate direction of arrival estimation. In L1-SVD the wideband signal is first decomposed into several narrowband components and each frequency band is treated independently and all the result are integrated to obtain the final result, but due to this it incorporates heavy computational load. While W-CMSR method CMSR processes the incident wideband signals holistically and do not decompose them in the frequency domain, thus it does not get into the trouble of integrating narrowband signal components to obtain the final wideband DOA estimates. By turning to the sparse signal representation framework, we are able to achieve super-resolution without the need for a good initialization, without a large number of time samples, and with lower sensitivity to SNR and to correlation of the sources. The topic of sparse signal representation has evolved very rapidly in the last decade, finding application in a variety of problems, including image reconstruction and restoration [8,9].
The simplest version of the sparse representation problem without noise is to find a sparse \( X \in \mathbb{C}^N \) given \( Y \in \mathbb{C}^M \) which are related by \( Y = AX \), with \( M \times N \). The matrix \( A \) is known. The assumption of sparsity of is crucial since the problem is ill-posed without it (\( A \) has a nontrivial null-space). An ideal measure of sparsity is the count of nonzero entries, which is denoted by \( \| X \|_0 \) which we also call the L0-norm. Hence, mathematically, we must look for arg \( \min \| X \|_0 \) such that \( Y = AX \). A sparse representation problem with additive Gaussian noise takes the following form: \( Y = AX + n \). To extend L1-penalization to the noisy case, an appropriate choice of an optimization criterion \( \min \| X \|_1 \) is subject to \( \| Y - AX \|_1 \leq \beta \), where \( \beta \) is a parameter specifying how much noise we wish to allow.

III. Direction Of Arrival Estimation Via Covariance Matrix Sparse Representation

In W-CMSR, the lower left triangular elements of the covariance matrix are aligned to form a new measurement vector, and DOA estimation is then realized by representing this vector on an over-complete dictionary under the constraint of sparsity. The a priori information of the incident signal number is not needed in W-CMSR, and no spectral decomposition or focusing is introduced. Also in W-CMSR the half-wavelength spacing restriction in avoiding ambiguity can be relaxed from the highest to the lowest frequency of the incident wideband signals. Some of the advantages of W-CMSR techniques are [10]:

1) W-CMSR relies less on the a priori information of the incident signal number than the ordinary subspace-based methods.

2) No spectral decomposition or focusing is introduced, thus W-CMSR is immune to imperfect DOA pre-estimates, and will not run into the tremendous efforts of focusing matrix selection or non identical narrowband DOA estimate fusion.

3) The a priori information of signal spectrum can be exploited in W-CMSR to improve the performance of DOA estimation.

4) Well-designed array geometries help W-CMSR to obtain enhanced ability in separating even more simultaneous signals than sensors.

5) The half-wavelength spacing restriction in avoiding ambiguity is relaxed from the highest to the lowest signal frequency, just like what JLZA-DOA has achieved.

In general the separation between the sensors is kept sparse that is half wavelength of highest frequency, so that the sensor noise is spatially uncorrelated, but in the case of W-CMSR this limitation is been relaxed from half wavelength of highest frequency to lowest frequency. Most of the previous methods decompose the incident wideband signals into narrowband components, and then realize wideband DOA estimation with incoherent or coherent techniques. However, there are two significant disadvantages within this kind of methods. First, DOA pre-estimates of the incident signals are required for spectral focusing, and the precision of those pre-estimates largely influences the performance of DOA estimation. Second, they need the a priori information of the incident signal number which may not be available, especially in the non-cooperative scenarios [11].

Suppose that ‘K’ wideband signals impinge onto an M-element array from the directions of \( \theta_1, \theta_2, \ldots, \theta_K \) respectively and ‘N’ snapshots are collected, the snapshot at time is given by:

\[
x(t) = \sum_{k=1}^{K} S_k(t + \tau_{k,t}) + v(t), \ldots, \sum_{k=1}^{K} S_k(t + \tau_{M,k}) + v_M(t)
\]

Where,

\( S_k(t) \) – Waveform of the Kth signal.

\( T_{m,k} \) - Propagation delay of Kth signal from Mth sensor.

\( V_m \) - Additive noise at Mth sensor.

The incident signals and the additive noise are assumed to be all Gaussian distributed and mutually independent. Thus the waveforms of the ‘K’ independent signals satisfy:

\[
E[S_k(t)S_{k2}(t)] = 0
\]
\[ E[S_k(t)S^*_k(t)] = \eta_k r_{k1}(t_1 - t_2) \delta(k_1 - k_2) \]

(5)

Where

\[ \eta_k \rightarrow \text{Power of } k^{th} \text{ signal} \]

\[ r_k(\tau) \rightarrow \text{Unified correlation} \]

\[ = E[S_k(t + \tau)S^*_k(t)] \]

\[ \eta_k \]

(6)

It satisfy \( r_\kappa(\tau) = 0 \) (we unify it to make this correlation function immune to signal power.)

### A. Direction Of Arrival estimation Via W-CMSR

The wideband DOA estimation method to be presented in this section bases on the correlation functions of the wideband signals, which can be derived from the array output covariance matrix. The perturbation-free covariance matrix of the wideband array output is given below [11]:

\[
R = \begin{bmatrix}
\sum_{k=1}^{K} \eta_k r(\tau_{2,k} - \tau_{1,k}) & \sum_{k=1}^{K} \eta_k r(\tau_{3,k} - \tau_{1,k}) & \cdots & \sum_{k=1}^{K} \eta_k r(\tau_{M,k} - \tau_{1,k}) \\
\sum_{k=1}^{K} \eta_k r(\tau_{2,k} - \tau_{1,k}) & \sum_{k=1}^{K} \eta_k r(\tau_{3,k} - \tau_{1,k}) & \cdots & \sum_{k=1}^{K} \eta_k r(\tau_{M,k} - \tau_{2,k}) \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{k=1}^{K} \eta_k r(\tau_{M,k} - \tau_{1,k}) & \sum_{k=1}^{K} \eta_k r(\tau_{M,k} - \tau_{2,k}) & \cdots & \sum_{k=1}^{K} \eta_k + \sigma^2_n
\end{bmatrix}
\]

(7)

Where, \( \sigma^2_n \) is the variance of additive noise.

The directions of the incident signals can be derived from the correlation family of \( \{ r(\tau_{M,k} - r_{1,k}) \}_{m1,m2=1,...,M} \). Various array geometries can be employed to the sensors and direction of arrival can be found for each array geometry.

## (i) General W-CMSR(GLA)

The covariance matrix of the array output is conjugate symmetric, thus its upper right triangular elements can be represented by the lower left triangular ones. And as the main diagonal elements are contaminated by the unknown noise variance we slide over them and align the lower left triangular elements column-by-column to obtain the following measurement vector [12]

\[ Y = [R_{2,1}, \ldots, R_{M,1}, R_{3,2}, \ldots, R_{M,2}, R_{M-1,3}, \ldots, R_{M,M-2}, R_{M,M-1}]^T \]

(8)

Where, \( R_{m1,m2} \) denotes the \((m1,m2)\)th element of \( R \).

This vector can be decomposed into \('K'\) components as

\[ Y = \sum_{k=1}^{K} Y_k \]

(9)

Where,

\[ Y_k = [r(\tau_{2,k} - \tau_{1,k}), \ldots, r(\tau_{M,k} - \tau_{1,k}), \ldots, r(\tau_{M,k} - \tau_{M-1,k})]^T \]

(10)

Those signal components depend on the unified correlation function of the incident signals and their propagation delays along the array, which are directly related to the signal directions. Thus, the signal directions can be estimated from if the signal components can be separated. Denote the propagation delay of a signal at direction \( \theta \) thus
\[ Y^{(\theta)} = [r^{(\theta)}(\tau_2) - r^{(\theta)}(\tau_1)], ..., r^{(\theta)}(\tau_M) - r^{(\theta)}(\tau_{M-1})]^T \]  

(11)

If one discretizes the \([-90^\circ, 90^\circ]\) scope \(\Delta\) then \(\phi = [-90^\circ, 90^\circ + \Delta, ..., 90^\circ] \) with interval of \(\Delta\), then thus \(Y\) can be rewritten in an over-complete form as

\[ Y = Y(\phi)n \]  

(12)

Where ‘n’ is a sparse vector and \(\{Y(\phi) = y^{(\theta)}| \theta \in \phi\}\), Based on the this model, the following constrained sparsity-enforcing objective function can be introduced for wideband DOA estimation-

\[ \hat{n} = \arg\min_{n} \| n \|_0, \text{ subject to } Y = Y(\phi)n \]  

(13)

Where \(\| . \|_0\)→L0-norm, It corresponds to number of non zero elements.

\[ \text{Argmin} \rightarrow \text{Minimizes the number of non-zero elements to get the direction of signals.} \]

The formulation of the over-complete dictionary \(Y(\phi)\) plays an important role during the DOA estimation process and the dictionary atoms depend on the correlation function. Further two techniques are used to find the correlation function:

1) It exploits the a priori information of the signal spectrum- In some scenarios, the correlation function can be computed from the prior information of signal modulation. For example, the amplitude of the unified correlation functions of phase- (PSK) signals own a triangular shape approximately, with its three vertexes located at \((\pm \frac{\pi}{2}, 0)\) and \((0,1)\) thus if the code rate is given or estimated with other methods, the unified correlation function can be computed straightforwardly as follows:

\[ r(\tau) = \begin{cases} 1 - |B| e^{j2\pi f \tau} & |\tau| \leq \frac{1}{B} \\ 0 & |\tau| > \frac{1}{B} \end{cases} \]  

(14)

2) If no such prior information is available with us than correlation is found out using array output only in following manner-

\[ r(\tau) = \frac{1}{P} \int P(\omega) e^{j\omega \tau} d\omega \]  

(15)

B. Uniform Linear Array (ULA)

When more than one pair of the array sensors are equally spaced, repeated elements exist in. Those elements increase the problem dimension without providing any innovative information.

IV. Conclusion

This paper survey various direction of arrival estimation techniques. The paper mainly focus on the sparse representation techniques for the estimation of direction of arrival. The W-CMSR technique seems to be better than the other existing techniques. In future, the W-CMSR technique can be simulated using MATLAB and results can be compared with other existing techniques.

References


